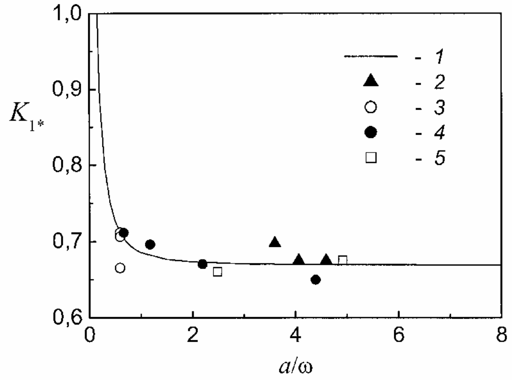

Fig. 1 Effect of parameter κ on the constant K_{1*} at $Pr = 0.71$. 1—present predictions at $T_w = \text{const}$. Experiments[4]: 2—$Re_j = 2.47 \times 10^4$, $Re_d = 8.56 \times 10^5$, $h_j/d_j = 2$, 4, and 6; 3—$Re_j = 6.8 \times 10^3$, $Re_d = 1.584 \times 10^6$, $h_j/d_j = 2$, 4, and 6. Experiments[5]: 4—$Re_j = 6.8 \times 10^3$, $Re_d = 2.14 \times 10^5$, 4.28×10^5, 8×10^5, and 1.412×10^6, $h_j/d_j = 2$; 5—$Re_j = 2.47 \times 10^4$, $Re_d = 8 \times 10^3$, and 1.584×10^6, $h_j/d_j = 2$.

Comparisons with Experiments

For coaxial impingement of jets with $d_j < d$, Eqs. (12) and (13) are valid only inside the stagnation region at $r \leq d_j/2$. In this case value d_j is used instead of d in both Nu_d and Re_ω. The disk surface in experiments[4,5] in the stagnation region was practically isothermal ($n^* = 0$). Using the definition of parameter A, we have $\kappa = ARe_j/Re_\omega$. In accordance with Eq. (2), parameter A takes values $A = 1.29$, 1.14, and 1.01 for $h_j/d_j = 2$, 4, and 6, respectively. These values allow reaching good agreement of predictions with experiments[4,5] for Nu_d at $Re_j = 2.47 \times 10^4$. At $Re_j = 6.8 \times 10^3$, experiments do not confirm the tendency of decrease in Nu_d with growing h_j/d_j. The best agreement with experiments at $Re_j = 6.8 \times 10^3$ yields the value $A = 1.12$.

Comparisons of simulations with experiments are shown in Fig. 1. Predictions agree well with experiments clearly exhibiting the tendency of increase with decreasing values of κ. It is apparent also that parameter $K_{1^*} = K_1 \cdot (1 + \kappa^{-1})^{1/2}$ is practically constant for $\kappa = 1.5 \ldots \infty$. It means that whenever the parameter κ exceeds the threshold value of 1.5, impingement heat transfer of a rotating disk depends only on Re_j and is independent of the speed of rotation.

Three experimental points fall out of this generally good agreement. Too high experimental value of K_{1^*} at $Re_j = 2.47 \times 10^4$, $Re_d = 8.56 \times 10^5$, and $h_j/d_j = 6$ ($\kappa = 3.6$) is probably caused by the too low value $A = 1.01$ used in recalculation of experimental data for Nu_d. Too low experimental value of K_{1^*} at $Re_j = 6.8 \times 10^3$, $Re_d = 2.12 \times 10^5$ and $h_j/d_j = 2$ ($\kappa = 4.4$) is probably caused by the fact that we used the constant value $A = 1.12$ in recalculation of Nu_d at $Re_j = 6.8 \times 10^3$ and varying Re_ω, while rotation can affect A at low values of Re_j. Too low value of K_{1^*} at $Re_j = 6.8 \times 10^3$, $Re_\omega = 1.584 \times 10^6$, and $h_j/d_j = 4$ ($\kappa = 0.6$) is probably explained by experimental inaccuracy. To clarify the dependence of A on h_j/d_j and Reynolds numbers Re_j and Re_ω, an additional experimental research is needed.

Conclusions

An integral method was developed, and an approximate analytical solution of the problem was derived at $n^* = -2 \ldots 4$. Maximal deviation of the approximate solution from the exact one is 2.4%. At $\kappa > 1.5$ heat transfer is dominated only by peculiarities of the impinging jet. The threshold value of κ is practically independent of Pr and exponent n^*. Present predictions agree well with experiments[4,5] in the vicinity of the stagnation point.

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Scale-Size Analysis of Heat and Mass Transfer Correlations

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Nomenclature

C	=	constant coefficient
D	=	characteristic length or hydraulic diameter, m
g	=	gravity acceleration, ms^{-2}
h	=	heat transfer coefficient, $Wm^{-2}K^{-1}$
K	=	transport coefficient, ms^{-1}
k	=	thermal conductivity, $Wm^{-1}K^{-1}$
L	=	macrolength, m
l	=	microlength, m
m	=	convective term exponent
q	=	diffusive term exponent
T	=	macrotime, s
t	=	microtime, s
U	=	mean flow velocity, ms^{-1}
α_M	=	mass diffusivity, m^2s^{-1}
α_T	=	thermal diffusivity, m^2s^{-1}
β	=	expansion coefficient at constant pressure, K^{-1}
ΔT	=	temperature difference, K
η	=	dynamic viscosity, Pa · s
λ	=	macrolength dimensional exponent

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ν = kinematic viscosity, $m^2 s^{-1}$
τ = macrotime dimensional exponent

Subscripts

M = mass
s = surface
T = thermal

Introduction

HEAT and mass transfer correlations for both free and forced convective flows express the functional relationship among dimensionless parameters in the form of power laws, where the exponents of the Reynolds, Grashof, Prandtl, and Schmidt numbers are often empirically determined for a large variety of geometries and flow conditions. In particular, the correlations for forced flows are usually expressed as

$$Nu = C Re^m Pr^q \tag{1a}$$

$$Sh = C Re^m Sc^q \tag{1b}$$

whereas in free convective flow correlations the Reynolds number is replaced by the Grashof number,

$$Nu = C Gr^m Pr^q \tag{2a}$$

$$Sh = C Gr^m Sc^q \tag{2b}$$

The dimensionless groups are detailed in Table 1. The Nusselt number can be put into the same form as the Sherwood number after introducing the thermal diffusivity $\alpha_T = k/\rho c_p$, so that

$$Nu = K_T D/\alpha_T \tag{3}$$

where $K_T = h/\rho c_p$ is the thermal energy transport coefficient.

This work presents an approach to the estimation of the Reynolds and Grashof numbers exponent based on the comparison of two different space- and timescales. In particular, the scale-size analysis allows the definition of an upper bound for laminar flows and a lower bound for turbulent flows.

Analysis

The dimensionless numbers appearing in correlations (1) and (2) are constructed with physical quantities that can be sorted into two groups, according to their length and time characteristic scales: 1) macroquantities or global quantities (such as the hydraulic diameter or the characteristic dimension D and the mean flow velocity U), which can be expressed in terms of a macrolength L and a macrotime T and 2) microquantities or molecular quantities (such as the thermal and the mass diffusivity and the kinematic viscosity), which can be expressed in terms of a microlength l and a microtime t.

This does not apply to the transport coefficients K_T and K_M (both of which have the dimensions of a velocity), so that splitting length and time into their micro- and macrocomponents is required:

$$[K_i] = \frac{[L]^{\lambda}[l]^{1-\lambda}}{[T]^{\tau}[t]^{1-\tau}}, \qquad i = M, T \tag{4}$$

Table 1 Dimensionless numbers

Symbol	Name	Description
Gr	Grashof	$g\beta(\Delta T)D^3/\nu^2$
Nu	Nusselt	hD/k
Pr	Prandtl	ν/α_T
Re	Reynolds	UD/ν
Sc	Schmidt	ν/α_M
Sh	Sherwood	$K_M D/\alpha_M$

where λ and τ are the characteristic exponents of the macrolength and the macrotime in the transport coefficient, respectively.

The structure of transport coefficients in turbulent flows is different from that in laminar flows: In fact, macroquantities are relevant in turbulent flows, which are dominated by inertia forces, so that their exponents in Eq. (4) must be larger than those of the corresponding microquantities:

$$\lambda > 1 - \lambda \tag{5a}$$

$$\tau > 1 - \tau \tag{5b}$$

On the other hand, microquantities become significant in laminar flows, which are dominated by diffusion processes, and consequently their exponents in Eq. (4) must be larger than those of the corresponding macroquantities:

$$\lambda < 1 - \lambda \tag{6a}$$

$$\tau < 1 - \tau \tag{6b}$$

The distinction between global quantities and molecular quantities unfolds the scale-dependent dimensional groups inside dimensionless numbers:

$$[Nu] = [Sh] = \frac{[L]^{1+\lambda}[l]^{-(1+\lambda)}}{[T]^{\tau}[t]^{-\tau}} \tag{7a}$$

$$[Re] = \frac{[L]^2[l]^{-2}}{[T][t]^{-1}} \tag{7b}$$

$$[Gr] = \frac{[L]^4[l]^{-4}}{[T]^2[t]^{-2}} \tag{7c}$$

Of course, the Prandtl and the Schmidt numbers cannot be given such a representation because they are a function of molecular quantities only.

By substituting Eqs. (7) into Eqs. (1) and (2), for convective, forced flows one obtains

$$m = (1 + \lambda)/2 = \tau \tag{8}$$

whereas for convective free flows

$$m = (1 + \lambda)/4 = \tau/2 \tag{9}$$

The constraint on the Reynolds number exponent in forced convective flows can be determined separately for turbulent or laminar flows by combining Eqs. (8) and (5) or Eqs. (8) and (6), respectively; similarly, the constraint on the Grashof number exponent in free convective flows can be determined from the combination of Eqs. (9) and (5) for turbulent flows and of Eqs. (9) and (6) for laminar flows. The results are reported in Table 2.

Table 2 Values of the constraints on the exponents of the Reynolds and of the Grashof numbers

	Constraint	
Exponent	Turbulent	Laminar
Forced convective flow correlation		
λ	>0.5	<0.5
τ	>0.5	<0.5
m [Eqs. (1a) and (1b)]	>0.75	<0.5
Free convective flow correlation		
λ	>0.5	<0.5
τ	>0.5	<0.5
m [Eqs. (2a) and (2b)]	>0.375	<0.25

Discussion

The values assigned to the exponent m in most of the correlations available in the open literature[1,2] are coherent with the constraints determined by means of the scale-size analysis. In forced laminar convection in circular pipes, it is well known that the analytical solution of the energy equation yields a constant Nusselt number, so that $m = 0$. In the entry region, the Sieder and Tate correlation,[3] where $m = \frac{1}{3}$, is often recommended:

$$Nu = 1.86[RePr/(L/D)]^{\frac{1}{3}}(\eta/\eta_s)^{0.14} \tag{10}$$

In Eq. (10), η and η_s are evaluated at the mean and at the surface temperature, respectively.

In forced turbulent convection, we have $m = 0.8$ in the widely used Dittus–Boelter heat transfer correlation for fully developed flow in a circular tube,[4]

$$Nu = 0.023Re^{0.8}Pr^{n} \tag{11}$$

In free laminar convection on a flat vertical plate, one obtains for the Nusselt number analytical solutions of the form:

$$Nu = \tfrac{4}{3}(Gr/4)^{\frac{1}{4}}g(Pr) \tag{12}$$

where $g(Pr)$ is a function of the Prandtl number (see Refs. 5 and 6).

The apparent disagreement that can be noticed in some cases is because the flow regime is not the same in the whole region where the heat or the mass transfer occurs. For instance, this is the case of the flow around a cylinder, where the flow boundary layer changes from laminar to turbulent along the cylinder perimeter. Thus, the exponents of the dimensionless numbers appearing in the correlation are involved in the integration of local values; for example, a common heat transfer correlation for external flow across a pipe, valid for $10^3 < Re < 2 \times 10^5$, is

$$Nu = 0.25Re^{0.6}Pr^{0.38} \tag{13}$$

where the value of the exponent m is inconsistent with both the constraint for turbulent flows ($m > 0.75$) and with that for laminar flows ($m < 0.5$).

Conclusions

An approach based on two kinds of space- and timescales is used to determine bounds on the exponents of the Reynolds number and the Grashof number contained in the heat and mass transfer correlations for free and forced convective flows. Upper bounds are obtained for laminar flow and lower bounds for turbulent flow. The exponent of the Prandtl number is independent of the exponents of micro- and macroscales, so that no bounds can be determined through the two-scale dimensional analysis.

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